This set contains three pages (beginning with this page)
All questions must be answered
Question 1 weighs 25 %, while questions 2 and 3 each weigh 37,5 %. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO AUGUST 17, 2012 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model with monopolistic competition and sticky prices, a monetary policy implementing the Friedman rule is optimal as it eliminates any relative demand distortions.

A False. Implementing the Friedman rule, lead to a deflation rate equal to the real rate of interest. With sticky prices of the Calvo type, some firms will not be able to lower their prices in proportion to the aggregate price fall. Hence, there will be sticky-price induced price dispersion, and thus relative demand distortions in the economy. The optimal rate of inflation in the simple New-Keynesian model is therefore one that makes the inability to adjust prices irrelevant: zero inflation.

(ii) Consider the model of Barro and Gordon, where output, y, is given by $y = \pi - \pi^e + \varepsilon$, where π is inflation, π^e is inflation expectations and ε is a supply shock. Social welfare is given by $V = -(y-k)^2 - \pi^2$, $k > 0$. Delegating monetary policy conduct to a "conservative" central banker with utility function $V^c = -(y-k)^2 - (1+\delta)\pi^2$, $\delta > 0$, is disadvantageous if the variance of ε is sufficiently high.

A **False.** In the Barro and Gordon model, the discretionary equilibrium is characterized by an inefficiently high inflation rate, the so-called inflation bias. The policy reaction to the supply shock, on the other hand, is efficient. By delegation to a "conservative" central banker, the average inflation rate will be brought down, but shock stabilization is distorted (inflation becomes suboptimally stable, and output too unstable). This could at first glance suggest that if shock stabilization is sufficiently important, that if the variance of ε is sufficiently high, then a conservative central banker is disadvantageous. This reasoning is wrong, however, as the beneficial effects of $\delta > 0$ are of first order (it mitigates an inefficiency), while the detrimental effects are of second order as shock stabilization is efficient at $\delta = 0$. Hence, some $\delta > 0$ is optimal. The very good answer will note that the optimal value of δ will decrease with the variance of the supply shock.

(iii) In the Lucas "islands" model, anticipated aggregate money shocks have real effects as agents — due to imperfect information — cannot distinguish between local money disturbances and aggregate money disturbances.

A **False.** In the Lucas "islands" model, it is *un*anticipated aggregate money shocks, which have real effects. When these arrive, agents do not know — due to imperfect information — whether the observed money shock is caused by local factors (in which case they should react) or by aggregate factors (in which case all prices will move proportionally, and no reaction is optimal). In performing the signal extraction, they optimally react somewhat as they assign some probability to the fact that the shock is local. How much they react will depend upon the relative variances of local and aggregate shocks (e.g., a high variance of local shocks makes it relatively likely that a shock is indeed local, and agents therefore respond relatively strong to the observed shock). If an aggregate shock is anticipated, agents know by assumption that the observed shock an aggregate shock (not affecting relative prices), so they optimally do nothing.

QUESTION 2:

Consider an infinite-horizon economy in discrete time, where the utility of the representative agent is given by

$$U = \sum_{i=0}^{\infty} \beta^i \left[\ln c_{t+i} + \ln\left(1 - n_{t+i}\right)\right], \qquad 0 < \beta < 1, \tag{1}$$

where c_t is consumption in period t, and n_t is employment. The economy is characterized by flexible prices and perfect competition in the goods and labor markets. Agents have perfect foresight and face the budget constraint

$$c_t + b_t + m_t \leq y_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t, \tag{2}$$

where y_t is real income, b_{t-1} denotes real government bond holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate, π_t is the inflation rate, m_{t-1} is real money holdings, and τ_t denotes real government transfers. Income (output) is produced with labor as only input:

$$y_t = n_t^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{3}$$

Purchases of consumption goods are subject to a cash-in-advance constraint:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{4}$$

(i) Agents maximize utility. Find the relevant first-order conditions characterizing the optimal choices of c_t, n_t, and m_t, and interpret them intuitively. [Hint: Use dynamic programming and express the value as a function of the state variables b_{t-1} and m_{t-1}; substitute out b_t by constraint (2), and let μ_t be the multiplier on (4).]

A Using the hint, the value function can be stated as

$$V(b_{t-1}, m_{t-1}) = \max_{c_t, n_t, m_t} \left\{ \begin{array}{c} \ln c_t + \ln(1-n_t) + \beta V(b_t, m_t) \\ -\mu_t \left[c_t - \dfrac{m_{t-1}}{1+\pi_t} - \tau_t \right] \end{array} \right\},$$

where one from the budget constraint (also using the production function) has that

$$b_t = n_t^{1-\alpha} + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - c_t - m_t.$$

Using this expression for b_t in the value function, the relevant first-order conditions follow as

$$\begin{aligned} 1/c_t - \beta V_b(b_t, m_t) - \mu_t &= 0, \\ -1/(1-n_t) + \beta V_b(b_t, m_t)(1-\alpha) n_t^{-\alpha} &= 0, \\ \beta V_m(b_t, m_t) - \beta V_b(b_t, m_t) &= 0. \end{aligned}$$

All these are interpreted as marginal gains in terms of, respectively, current consumption, leisure and money, being equal to marginal losses in terms of lost future wealth and/or current liquidity costs (of consumption due to the cash-in-advance constraint).

(ii) Use the envelope theorem to eliminate the partial derivatives of the value function, let $\lambda_t \equiv \beta V_b(b_t, m_t)$ where V is the value function and V_b denotes $\partial V(b_t, m_t)/\partial b_t$, and show that the steady state can be characterized by

$$\begin{aligned} 1/c^{ss} &= \lambda^{ss}(1+i^{ss}), \\ 1/(1-n^{ss}) &= \lambda^{ss}(1-\alpha)(n^{ss})^{-\alpha}, \\ \beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \end{aligned}$$

where superscript "ss" denotes steady-state values. Explain.

A The value function derivatives are, by application of the envelope theorem (implying that any effect of b_{t-1} and m_{t-1} on c_t, n_t and m_t cancel out by the first-order conditions), found as

$$\begin{aligned} V_b(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t)\frac{1+i_{t-1}}{1+\pi_t}, \\ V_m(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t)\frac{1}{1+\pi_t} + \mu_t\frac{1}{1+\pi_t}. \end{aligned}$$

Use the hint and define

$$\lambda_t \equiv \beta V_b(b_t, m_t).$$

The first of the value function derivatives can therefore be written as

$$\lambda_t = \beta\lambda_{t+1}\frac{1+i_t}{1+\pi_{t+1}}.$$

The second can be rewritten as

$$\begin{aligned} V_m(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1})\frac{1}{1+\pi_{t+1}} + \mu_{t+1}\frac{1}{1+\pi_{t+1}}, \\ V_b(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1})\frac{1}{1+\pi_{t+1}} + \mu_{t+1}\frac{1}{1+\pi_{t+1}}, \\ \beta V_b(b_t, m_t) &= \beta^2 V_b(b_{t+1}, m_{t+1})\frac{1}{1+\pi_{t+1}} + \beta\mu_{t+1}\frac{1}{1+\pi_{t+1}}, \\ \lambda_t &= \beta\frac{\lambda_{t+1}+\mu_{t+1}}{1+\pi_{t+1}}, \end{aligned}$$

where the second line uses the third of the first-order conditions. The first two first-order conditions can be rewritten as

$$1/c_t - \lambda_t - \mu_t = 0,$$

$$-1/(1-n_t) + \lambda_t(1-\alpha)n_t^{-\alpha} = 0.$$

Hence, one has

$$\begin{aligned}
1/c_t - \lambda_t - \mu_t &= 0, \\
-1/(1-n_t) + \lambda_t (1-\alpha) n_t^{-\alpha} &= 0, \\
\lambda_t &= \beta \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}}, \\
\lambda_t &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}},
\end{aligned}$$

which in steady state becomes:

$$\begin{aligned}
1/c^{ss} - \lambda^{ss} - \mu^{ss} &= 0, \\
-1/(1-n^{ss}) + \lambda (1-\alpha) (n^{ss})^{-\alpha} &= 0, \\
\beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \\
\beta^{-1} &= \frac{1+\mu^{ss}/\lambda^{ss}}{1+\pi^{ss}}.
\end{aligned}$$

This is readily reformulated as

$$\begin{aligned}
1/c^{ss} &= \lambda^{ss} (1+i^{ss}), \\
1/(1-n^{ss}) &= \lambda^{ss} (1-\alpha) (n^{ss})^{-\alpha}, \\
\beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}},
\end{aligned}$$

as required.

(iii) Derive steady-state employment as a function of the nominal interest rate. [Hint: Use the economy's resource constraint $y_t = c_t$.] Explain.

A Combining the first two steady-state requirements, on can express employment and consumption as a function of i^{ss}:

$$c^{ss}/(1-n^{ss}) = \frac{(1-\alpha^{ss})(n^{ss})^{-\alpha}}{1+i^{ss}}.$$

Then use the hint to express consumption as a function of employment, $c^{ss} = (n^{ss})^{1-\alpha}$. One thus gets

$$\begin{aligned}
\frac{(n^{ss})^{1-\alpha}}{1-n} &= \frac{(1-\alpha)(n^{ss})^{-\alpha}}{1+i^{ss}}, \\
\frac{n^{ss}}{1-n^{ss}} &= \frac{1-\alpha}{1+i^{ss}},
\end{aligned}$$

and thus

$$n^{ss} = \frac{1-\alpha}{i^{ss}+2-\alpha}.$$

One sees that employment is a decreasing function of the nominal interest rate. Monetary superneutrality thus fails in the model, as different inflation rates leads to different nominal interest rates, and thus different employment and output levels. The intuition is that consumption is "taxed" by the cash-in-advance constraint for positive nominal interest rates, while leisure is not. An increasing nominal interest rate thus makes consumption relatively more expensive than leisure, and agents substitute away from consumption and supply less labor.

(iv) Derive the monetary policy that generates the utility-maximizing solution for employment. Explain.

A The optimal monetary policy is one that alleviates the distortionary nature of the cash-in-advance constraint. Here, this will be one that implements the Friedman rule. I.e., $i^{ss} = 0$. Hence, the optimal employment level is

$$n^{ss} = \frac{1-\alpha}{2-\alpha}.$$

Technically, this can also be seen by finding the utility-maximizing employment level in steady state:

$$\max_{n^{ss}} \left\{ \ln\left[\left(n^{ss}\right)^{1-\alpha} \right] + \ln\left(1-n^{ss}\right) \right\}$$

The first-order condition is

$$\frac{(1-\alpha)}{n^{ss}} = \frac{1}{1-n^{ss}}$$

yielding

$$n^{ss} = \frac{1-\alpha}{2-\alpha}.$$

This is employment in the cash-in-advance economy only for $i^{ss} = 0$.

QUESTION 3:

Consider the following model for output and inflation determination in a closed economy:

$$y_t = \theta y_{t-1} - \sigma\left(i_{t-1} - \mathrm{E}_{t-1}\pi_t\right) + u_t, \qquad 0 < \theta < 1, \quad \sigma > 0, \tag{1}$$

$$\pi_t = \pi_{t-1} + \kappa y_t + \eta_t, \qquad \kappa > 0, \tag{2}$$

where y_t is log of output in period t, i_t is the nominal interest rate (the monetary policy instrument), π_t is the inflation rate, u_t and η_t are independent, mean-zero, serially uncorrelated shocks. E_j is the rational expectations operator conditional on information up to and including period j. It is assumed that $\sigma\kappa < 1$.

(i) Discuss equations (1) and (2), with emphasis on the monetary transmission mechanism and the stability properties in absence of policy intervention (only a verbal discussion is required).

A The Main points are that there are lagged effects of output and inflation in the IS and Phillips curves, respectively. Moreover, policy takes effect on demand with a one-period lag. In absence of policy intervention, the model is unstable, as, e.g., a positive demand shock will increase demand, subsequently inflation, subsequently lower the real interest rate, and then further expand output, increase inflation, and so on.

The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2}\mathrm{E}_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^2, \qquad 0 < \beta < 1.$$

(ii) Find the optimal interest-rate rule for i_t as a function of π_t and y_t. (Hint: Treat $\mathrm{E}_t y_{t+1} \equiv y_{t+1} - u_{t+1}$ as the policy instrument, and solve the maximization problem by dynamic programming treating π_t as the state variable. That is, find the optimal policy as $\mathrm{E}_t y_{t+1} = B\pi_t$, where B is a parameter to be found, and use (1) and (2) to derive the associated nominal interest rate.)

A Using the hint, the relevant value function becomes

$$\begin{aligned} v(\pi_t) &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t \left\{ -\frac{1}{2}\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right)^2 + \beta v\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right) \right\}, \\ &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t \left\{ \begin{array}{l} -\frac{1}{2}\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right)^2 \\ +\beta v\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \end{array} \right\}. \end{aligned}$$

The first-order condition is

$$\begin{aligned} & -\mathrm{E}_t \kappa\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \\ & +\mathrm{E}_t \beta\kappa v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \\ = \; & 0, \end{aligned}$$

$$-\mathrm{E}_t \kappa\left(\pi_t + \kappa\mathrm{E}_t y_{t+1}\right) + \mathrm{E}_t \beta\kappa v'\left(\pi_t + \kappa\mathrm{E}_t y_{t+1}\right) = 0,$$

$$-\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right)+\beta v^{\prime}\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right)=0.$$

Using the Envelope Theorem one gets:

$$v^{\prime}\left(\pi_t\right)=-\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right)+\beta v^{\prime}\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right).$$

So, $v^{\prime}\left(\pi_t\right)=0$. Hence, $\mathrm{E}_t y_{t+1}=-\frac{1}{\kappa}\pi_t$, showing that $B=-\kappa^{-1}<0$. We also have that

$$\begin{aligned}
\mathrm{E}_t y_{t+1} &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t\left[\pi_t+\kappa y_{t+1}\right]\right), \\
\mathrm{E}_t y_{t+1}(1-\sigma\kappa) &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t \pi_t\right),
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}_t y_{t+1}(1-\sigma\kappa) &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t \pi_t\right) \\
\mathrm{E}_t y_{t+1} &= \frac{\theta}{(1-\sigma\kappa)} y_t-\frac{\sigma}{(1-\sigma\kappa)}\left(i_t-\mathrm{E}_t \pi_t\right).
\end{aligned}$$

So, the interest rate rule follows from

$$-\frac{1}{\kappa}\pi_t=\frac{\theta}{(1-\sigma\kappa)} y_t-\frac{\sigma}{(1-\sigma\kappa)}\left(i_t-\mathrm{E}_t \pi_t\right),$$

$$-\frac{(1-\sigma\kappa)}{\kappa}\pi_t=\theta y_t-\sigma\left(i_t-\pi_t\right),$$

$$\sigma i_t=\left[\sigma+\frac{(1-\sigma\kappa)}{\kappa}\right]\pi_t+\theta y_t,$$

as

$$i_t=\left[1+\frac{(1-\sigma\kappa)}{\sigma\kappa}\right]\pi_t+\frac{\theta}{\sigma} y_t.$$

(iii) Comment on the coefficient on π_t in the optimal interest rate rule, with special emphasis on how its value affects the stability properties of the model.

A The main issue is that the coefficient is greater than one. Hence, it is an active Taylor-type rule, such that any rise in inflation is met by a larger increase in the nominal interest rate. This increases the real interest rate, and will serve to contract output and thus reduce inflation. Hence, it serves a stabilizing role.

(iv) Discuss how the coefficients on π_t and y_t in the optimal interest rate rule depend on the underlying parameters of the model. and discuss whether the parameters reveal anything about the "strict" inflation-targeting preferences of the central bank.

A It can be seen that the structural parameters σ and κ reduce the inflation coefficient. This is because when these values are lower, a smaller nominal interest rate response is needed to stabilize inflation (as demand is more sensitive and inflation is more sensitive to demand). Furthermore it is observed that output increases will lead to nominal interest rate changes, even though the central bank is conducting strict inflation targeting. The reason being that output changes provides information about inflation one period ahead (as long as there is output inertia; i.e., as $\theta > 0$). Hence, the parameter values, and the variables in the interest rate rule, tell nothing about the preferences of the central bank. From the curriculum, it is known that a model with a flexible inflation-targeting bank yields the same form of the optimal rule.